Subatomic Physics 2013/20014

Exam Subatomic Physics 2013/2014
Monday, 27 - January - 2014, 14:00 - 17:00

70 points

Please mark your name and your student number on every sheet.

You receive 6 A4 pages:

- pages 2, 3 and 4 contain 6 topics from the lecture.
- page 5 gives relevant Constants, Relations and Conversion Factors
- Note: For some of the questions different approaches are possible, such that you may not necessarily need all of the given constants and equations. The final results are sufficient, if given correct to 2 significant figures (2 leading digits).
- There is one evaluation form which you are kindly askeded to fill in and return with your answers.

Groningen, January 27, 2014
Best wishes for your exam,
Klaus Jungmann

1 Exam

1.1 Our Sun (15 points)

At present the sun has a mass of $M_S \approx 2 \cdot 10^{30}$ kg (which is about $3.3 \cdot 10^5$ earth masses M_E). The solar surface is composed of 71% hydrogen (H), 27% helium (^{4}He), and 2% heavier elements (percentage with respect to mass). The sun radiates energy at a total power $P_S = 4 * 10^{26} W$. The main energy producing mechanism in the center of the sun is the so called *proton-proton cycle*:

(i)	$p + p \rightarrow d + e^+ + \nu_e$	$+E_1$;	$E_1 = +0.42 MeV$
(ii)	$p + d \rightarrow {}^3\text{He} + \gamma$	$+E_2$;	$E_2 = +5.49 MeV$
(iii)	${}^3\text{He} + {}^3\text{He} \rightarrow p + p + \alpha$	$+E_3$	
(iv)	$e^+ + e^- \rightarrow 2\gamma$	$+E_4$;	$E_4 = +1.02 MeV$
(v)	The sum of one full cycle is hence:		
	$4p \rightarrow \alpha + 2e^+ + 2\nu_e + 2\gamma$	$+E_5$	

Let's assume that the sun was formed in a very short period of time compared to its age and that the atoms it may have collected from the universe due to gravitation since is negligible. Let's assume further it runs constantly since its beginning.

(a) Which interactions are responsible for every of the steps (i) to (iv) in the *proton-proton cycle*? (3 points)

(b) Please draw the Feynman diagrams for the processes (i) to (iv) at the quark level. (3 points)

(c) How much energy is released in step (iii) (calculate E_3.), and in one full cycle (calculate E_5)? (3 points)

(d) How much hydrogen is converted into helium every second? (2 points)

(e) What percentage of hydrogen has been used up since the sun started $5 \cdot 10^9$ years ago? (2 points)

(f) How much mass did the sun loose since its birth $5 \cdot 10^9$ years ago? Compare this with the mass of the earth. (2 points)

1.2 Wave equations (10 points)

The Schroedinger equation is not relativistically co-variant.

(a) Which two relevant equations exist to improve on this problem? (3 points)

(b) Which particles are described by each of the equations? (3 points)

(c) The basic wave equations are first principles and cannot be derived from anything more fundamental, yet. Nevertheless, one can motivate them just like one can motivate the Schroedinger equation. Can you give the line of reasoning for this? (4 points)

1.3 Neutrino experiments (10 points)

The nature of the neutrinos is an urgent issue in modern physics. Big detectors are operated or are being set up to detect neutrinos. Among them are ICE-CUBE project and KM3NET. These detectors use 1 km^3 of antarctic ICE and sea water, respectively, as active detector medium. In these experiments, photomultipliers are deployed in the ice near the South Pole, respectively the Mediterranian water. These multipliers detect Cherenkov light from reaction products which emerge when cosmic neutrinos react with the atoms in the detectors.

(a) Give an example of a reaction that could be used to detect a muon neutrino (ν_μ) respectively a muon antineutrino($\overline{\nu_\mu}$). (2 points)

(b) Which of the fundamental interactions is needed for a (anti-)neutrino reaction? (1 point)

(c) Can these detectors be used to detect neutrinos from tritium decay (maximum β-decay energie $\approx$ 17 keV?) Give one or two sentences of reasoning. (3 points)

(d) At the highest up to date observed energies (about 10^{21}eV) the cross section for neutrino reactions is estimated to be of order 10^{-31} cm^2 to $10^{-34}cm^2$. How large must the flux of neutrinos at such energies be at minimum in order to observe on average at least one event per day in the ICE-CUBE or KM3 detectors? (4 points)

1.4 Weak Interactions (10 points)

The weak eigenstates and the mass/flavour eigenstates of quarks are known to be different.

(a) How are they related to each other? Explain briefly. (2 points)

(b) What is the situation for leptons? Explain briefly. (2 points)

(c) What would change, if somebody would observe in future the process $\mu^+ \rightarrow e^+ + \gamma$. (2 points)

(d) Give one example each of a (i) leptonic, (ii) semileptonic and (iii) non-leptonic weak interaction process. Draw the corresponding Feynman diagrams and explain how the coupling constants differ, which describe the vertices. (4 points)

1.5 Allowed and Forbidden Processes (15 points)

Which of the following processes are allowed and which are forbidden. Please give reasons for your judgment. For allowed processes give the (dominat) interaction.

(a) $e^+e^- \to \pi^+ + \pi^- + \pi^0 + \pi^- + \pi^+$ (1.5 points)

(b) $\overline{\nu_\mu} + p \to \mu^+ + n$ (1.5 points)

(c) $\nu_e + p \to e^+ + \pi^0 + \Lambda^0$ (1.5 points)

(d) $e^- + e^- \to \mu^- + \mu^-$ (1.5 points)

(e) $^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + e^- + e^-$ (1.5 points)

(f) $p \to \pi^+ + e^- + e^+ + \gamma$ (1.5 points)

(g) $\mu^- \to e^- + e^+ + e^-$ (1.5 points)

(h) $J/\Psi(2^1S_0) \to J/\Psi(1^1S_0) + 3\gamma$ (remember: $J/\Psi = (c\overline{c})$ bound state) (1 point)

(i) $p + \overline{p} \to b\overline{b}$ (1.5 points)

(h) $p + {}^8\mathrm{Be} \to {}^8\mathrm{Be} + n + \pi^+ + \pi^- + \pi^+$ (1.5 points)

1.6 Radioactivity (10 points)

One Curie (Ci) used to be the unit for (radio)-activity. It had been defined as the α-decay activity (for a certain α energy) of 1 g of the very long living $^{226}_{88}Ra$ isotope (1 Ci $= 3.7 \cdot 10^{10}$ decays per second).

(a) What is the half-lifetime (T_{Ra}) of ^{226}Ra? (2 points)

(c) Please sketch the α energy spectrum. (1 point)

(b) ^{226}Ra decays into ^{222}Rn. Give the reaction equation and estimate the α-energy. Which of the different contributions to binding energy in the nuclei changes most in this α-decay? (4 points)

(d) ^{222}Rn lives 3.8 days ($\mathrm{T}_{Rn} = 3.3 \cdot 10^5$ seconds). Starting from a clean sample of only ^{226}Ra, how much ^{222}Rn is available in a day, in a month and in a year? (3 points)

2 Constants, Relations and Conversion Factors

2.0.1 Constants that could be of relevance

Speed of light	c	$2.998 \cdot 10^8$	m/s
Planck constant	h	$4.136 \cdot 10^{-24}$	GeVs
	$\hbar = \frac{h}{2\cdot\pi}$	$6.582 \cdot 10^{-25}$	GeV/c
Electron charge	e	$1.602 \cdot 10-19$	C
Electron mass	m_e	0.510998918(44)	MeV/c^2
Proton mass	m_p	938.272029(80)	MeV/c^2
Neutron mass	m_n	939.565360(81)	MeV/c^2
Deuteron mass	m_d	1875.61282(16)	MeV/c^2
Helium-3 mass	$m_{^3He}$	2809.41334(24)	MeV/c^2
^{226}Ra mass	$m_{^{226}Ra}$	226.025402	u
^{222}Rn mass	$m_{^{222}Rn}$	222.017570	u
Alpha particle mass	m_α	3727.37917(32)	MeV/c^2
Electron neutrino mass	m_{ν_e}	< 2.2	eV/c^2
Muon mass	m_μ	105.658369(9)	MeV/c^2
Charged Pion mass	$m_{\pi^\pm}$	139.57018(35)	MeV/c^2
Avogadro's number	N_A	$6.02214179(30) \cdot 10^{23}$	mol^{-1}

2.0.2 Relations

Mass of Atom (Bethe-Weizaecker):

$M(A,Z) = Nm_n + Zm_p + Zm_e - a_vA + a_sA^{2/3} + a_c\frac{Z^2}{A^{1/3}} + a_a\frac{(N-Z)^2}{4A} + \frac{\delta}{A^{1/2}}$
with
$a_v = 15.67MeV/c^2$
$a_s = 17.23MeV/c^2$
$a_c = 0.714MeV/c^2$
$a_a = 93.15MeV/c^2$
$\delta = 0$ (odd A) or -11.2 MeV/c^2 (Z and N even) or +11.2 MeV/c^2 (Z and N odd)

Schroedinger equation:

$$\frac{-\hbar}{i}\frac{\partial}{\partial t}\Psi(\vec{r},t) = [-\hbar^2/2m\vec{\nabla}^2 + V(\vec{r},t)]\Psi(\vec{r},t) \quad (1)$$

classical momentum-energy relation free particle

$$E = \frac{p^2}{2m} \quad (2)$$

relativistic momentum-energy relation free particle

$$E^2 = p^2c^2 + m_0^2c^4 \quad (3)$$

2.0.3 Conversion Factors

Electronvolt	eV	$1.60217653(14) \cdot 10^{-19}$	J
Tesla	T	$0.561 \cdot 10^{30}$	MeV/(c$^2\cdot$ C $\cdot$ s)
Kilogram	kg	$5.60958896(48) \cdot 10^{35}$	eV/c^2
barn	b	$1 \cdot 10^{-28}$	m^2
atomic mass unit	u	931.494061(21)	MeV/c^2

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